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J. Math. Anal. Appl. 367 (2010) 69–74

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

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A priori estimate for a quasilinear problem depending on the gradient

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A R T I C L E   I N F O

Article history:
Received 23 November 2009
Available online 23 December 2009
Submitted by C. Gutierrez

Keywords:
A priori estimates
Positive solutions
Strongly non-linear problems
m-Laplacian

A B S T R A C T

The main purpose of this paper is to establish a priori estimate for positive solutions of some superlinear, quasilinear elliptic equations where the nonlinearity depends on x, u, and ∇u. Our argument does not need a non-existence result for the limit problem obtained by the usual blow-up procedure. This work is related to a previous one by Ruiz (2004) [9].
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1. Introduction

The main goal is to establish a priori estimates for positive solution of the problem

$$\begin{cases} -\Delta_m u = f(x, u, \nabla u), & \text{for } x \in \Omega, \\ u(x) = 0, & \text{for } x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $1 < m < N$, the function $f : \Omega \times [0, +\infty) \times \mathbb{R}^N \to [0, +\infty)$ is continuous, and $\Omega \subset \mathbb{R}^N$ is a C^2 bounded domain. Here $\Delta_m u = \operatorname{div}(|\nabla u|^{m-2}\nabla u)$ denotes the m-Laplacian operator.

We will assume that for all $(x, u, \eta) \in \Omega \times [u_0, +\infty) \times \mathbb{R}^N$ we have

$$u^q - c_0|\eta|^\beta \leqslant f(x, u, \eta) \leqslant c_1 u^p + c_0|\eta|^\beta \tag{1.2}$$

where $c_0, u_0 > 0$, $c_1 \geqslant 1$, $m - 1 < q \leqslant p < m_* - 1$, and $m - 1 \leqslant \beta < pm/(p+1)$. The constant m_* denotes the Serrin exponent given by $m_* = \frac{m(N-1)}{N-m}$.

Observe that in the literature, a priori estimates are often obtained combining a blow-up technique with Liouville type results. For the case of the Laplace operator see for example [2–5]. Problems as (1.1) have been recently studied by many authors, see for example [1,8,9].

Our approach was inspired by the work of D. Ruiz [9] which studies Problem (1.1) for $q = p$. We treat Problem (1.1) for $q \leqslant p$. As in [9], the idea is to establish a priori estimates on the pair (u, λ) solving the problem

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[1] Supported by FONDECYT-Grant 1080500.
[2] Supported by FONDECYT-Grant 1080430.

0022-247X/$ – see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmaa.2009.12.030

$$\begin{cases} -\Delta_m u = f(x, u, \nabla u) + \lambda, & \text{for } x \in \Omega, \\ u(x) = 0, & \text{for } x \in \partial\Omega, \end{cases}$$

with $\lambda \geqslant 0$ and $u \in C^1(\overline{\Omega})$.

In [1,8,9], arguments by contradiction are used to obtain a priori estimates for Problem (1.1). In fact, in the case $p = q$ with $q \in (m-1, m_* - 1)$, is assumed that there exists a sequence of pairs $\{(u_n, \lambda_n)\}$, where $(u_n)_n$ is an unbounded sequence in the L^∞ norm and x_n is a point for which u_n attains its maximum. Thus the blow-up technique establishes gives rise to a solution of the problem

$$-\Delta_m u \geqslant u^q \tag{1.3}$$

which is defined on either $\mathbb{R}^N$ as the points x_n tend to an interior point of Ω or in the half-space as the points x_n tend to the boundary of Ω. Hence Liouville type results are necessary.

A Liouville type result about inequation (1.3) is shown in [8], namely, a non-existence result about positive solutions in all of the space. To the case of the half-space, see [7]. In order to overcome the half-space situation, C. Azizieh and P. Clément [1] assume that Ω is convex, that $1 < m \leqslant 2$, and that f depends only on u. Combining these assumptions with the moving plane method allows to prove that the sequence $(x_n)_n$ stays far away from the boundary $\partial\Omega$. A positive solution of Problem (1.3) in $\mathbb{R}^N$ is then obtained. This contradicts the Liouville type result in [8].

In [9], the blow-up technique centered at a certain point y_0 instead of x_n has recently been used. Harnack type inequalities are used there to compare the values of u_n at different points of Ω (see [11,12]). The limit problem thus obtained is defined in all of $\mathbb{R}^N$. It has again contradicted a Liouville type result. Note that the arguments above are true only for $p = q \in (m-1, m_* - 1)$ because in the case $q < p$ the limit problem is

$$-\Delta_m u \geqslant 0.$$

This does not imply a contradiction. However by using an adaptation of Ruiz's argument, we obtain L^γ estimates of the blow-up sequence (u_n) on a fixed ball, and arrive at a contradiction. Observe that our argument does not need Liouville type results. Finally note that, even in the variational case, that is, when f does not depend on ∇u, our result is new.

The paper is organized as follows. In Section 2 we give some preliminaries. In Section 3 we study a priori bounds. Finally in the last section we give an application to existence results.

2. Preliminaries

In this section, we state two known lemmas without proof which lead to the proof of our main result. We begin with a result about the weak Harnack inequality due to Trudinger [11].

Lemma 2.1. *Let u be a non-negative weak solution of the inequality*

$$-\Delta_m u \geqslant 0$$

in a domain $\Omega \subset \mathbb{R}^N$. Take $\gamma \in (0, m_* - 1)$ and $R > 0$ so that $B_{2R} \subset \Omega$. Then there exists a constant $C = C(N, m, \gamma)$ such that

$$\inf_{B_R} u \geqslant C R^{-N/\gamma} \|u\|_{L^\gamma(B_{2R})}.$$

The next result is a generalization of [12, Lemma 2.4] which is proved in [9].

Lemma 2.2. *Let u be a non-negative weak solution of the inequality*

$$-\Delta_m u \geqslant u^q - M|\nabla u|^\beta$$

in a domain $\Omega \subset \mathbb{R}^N$, where $q > m-1$ and $m-1 \leqslant \beta < mq/(q+1)$. Take $\gamma \in (0, q)$, and let B_{R_0} be a ball of radius R_0 so that B_{2R_0} is included in Ω. Then there exists a positive constant $C = (N, m, p, \alpha, d, R_0)$ such that, for all $0 < R \leqslant R_0$, we have

$$\int_{B_R} u^\gamma \leqslant C R^{(N-m\gamma)/(q+1-m)}.$$

3. A priori estimates

In this section, using a variant of the blow-up technique introduced in [9], we establish a priori estimates for the problem

$$\begin{cases} -\Delta_m u = f(x, u, \nabla u) + \lambda, & \text{for } x \in \Omega, \\ u(x) = 0, & \text{for } x \in \partial\Omega, \end{cases} \tag{3.4}$$

where λ is a non-negative parameter.

We first state that λ is dominated by the L^∞-norm of u.

Lemma 3.1. *Let u be a positive solution of Problem* (3.4). *Then there is a positive constant c_0 depending only on Ω such that*

$$\lambda \leqslant c_0\Big(\max_{x\in\overline{\Omega}} u\Big)^{m-1}. \tag{3.5}$$

Proof. Let u be a positive solution of Problem (3.4). When $\lambda = 0$, inequality (3.5) holds, and when λ is positive, we have

$$-\Delta_m u = f(x,u,\nabla u) + \lambda \geqslant \lambda, \quad \text{for all } x \in \Omega.$$

Let v be the positive solution of the problem

$$\begin{cases} -\Delta_m v = 1, & \text{for } x \in \Omega, \\ v(x) = 0, & \text{for } x \in \partial\Omega, \end{cases}$$

and take $w(x) = (\lambda/2)^{1/(m-1)} v(x)$. It follows that $-\Delta_m w = \lambda/2 < -\Delta_m u$ in Ω and that $u = w$ on $\partial\Omega$. Using the comparison lemma (see [10]), we see that $u \geqslant w$ in Ω. Hence

$$\max_{x\in\overline{\Omega}} w \leqslant \max_{x\in\overline{\Omega}} u.$$

Taking $c_0 = 2(\max_{x\in\overline{\Omega}} v)^{1-m}$ we complete the proof. $\square$

The following is our a priori estimate result.

Theorem 3.2. *Let u be a C^1 positive solution of Problem* (3.4)*. Suppose that condition* (1.2) *is satisfied. Then there exists a positive constant C such that, for all $x \in \Omega$, we have*

$$0 \leqslant u(x) + \lambda \leqslant C.$$

Proof. For otherwise, there would exist a sequence (λ_n, u_n) which would be a solution of Problem (3.4), such that $\lambda_n \geqslant 0$, $u_n > 0$, and $\|u_n\|_{L^\infty(\Omega)} + \lambda_n \to \infty$. By Lemma 3.1, we may assume that $\|u_n\|_{L^\infty(\Omega)} \to \infty$. Let $x_n \in \Omega$ be so that $u_n(x_n) = \|u_n\|_{L^\infty(\Omega)}$. We introduce the notation:

$$S_n = \|u_n\|_{L^\infty(\Omega)} \quad \text{and} \quad \delta_n = d(x_n, \partial\Omega).$$

The remaining part of the proof will depend on the following four subsidiary lemmas.

Lemma 3.3. *Suppose $x_n \to x_0 \in \Omega$. Define*

$$\widetilde{\delta}_n = \sup\big\{\delta\colon\ x \in B_\delta(x_n)\ \Rightarrow\ u_n(x) > S_n/2\big\}.$$

Then there exists $\widetilde{x}_n \in \Omega$ such that $d(x_n, \widetilde{x}_n) = \widetilde{\delta}_n$ and $u_n(\widetilde{x}_n) = S_n/2$.

Proof. It is not difficult to see that $\widetilde{\delta}_n$ is well defined, and that $0 < \widetilde{\delta}_n < \delta_n$. Note that $d(x_n, y) = \widetilde{\delta}_n$ implies $u_n(y) \geqslant S_n/2$.

Now suppose that the conclusion of this lemma is false, then for all $y \in \Omega$ such that $d(x_n, y) = \widetilde{\delta}_n$ we would have $u_n(y) > S_n/2$. Then according to the continuity of the solution and the compactness of the ball, there would be an $\varepsilon > 0$ so that $u_n(y) > S_n/2$ for all $y \in B_{\widetilde{\delta}_n+\varepsilon}(x_n)$ and $u_n(y) \geqslant S_n/2$, which is impossible by the definition of $\widetilde{\delta}_n$. $\square$

Lemma 3.4. *Suppose $x_n \to x_0 \in \Omega$. Then there exist a constant $c > 0$ and a constant $N_0 \in \mathbb{N}$ such that, for all $n \geqslant N_0$, we have*

$$0 < c < \widetilde{\delta}_n S_n^{(p+1-m)/m}$$

where $\widetilde{\delta}_n$ is defined in Lemma 3.3.

Proof. For otherwise, there would be a subsequence, which we denote the same way, so that

$$\widetilde{\delta}_n S_n^{(p+1-m)/m} \to 0$$

as $n \to +\infty$. Define

$$w_n(x) = S_n^{-1} u_n\big(x_n + S_n^{(m-1-p)/m} x\big).$$

Then w_n would satisfy

$$-\Delta_m w_m = \widetilde{f}(x, w_n, \nabla w_n), \quad \text{for all } x \in B_{\delta_n S_n^{(p+1-m)/m}}(0)$$

where $\widetilde{f}(x, w_n, \nabla w_n) = S_n^{-p}(f(x_n + S_n^{(m-1-p)/m}, S_n w_n(x), S_n^{(p+1)/m} \nabla w_n(x)) + \lambda_n)$. According to condition (1.2) and Lemma 3.1, for n sufficiently large, we would have

$$\widetilde{f}(x, w_n, \nabla w_n) \leqslant c_o w_n^p + M S_n^{-p+\frac{(p+1)\beta}{m}} |\nabla w_n|^\beta + \lambda_n S_n^{-p} \leqslant c_o w_n^p + |\nabla w_n|^\beta + 1.$$

Observe that, for n sufficiently large, we have

$$B_{\widetilde{\delta}_n S_n^{(p+1-m)/m}}(0) \subset B_1(0) \subset B_{\delta_n S_n^{(p+1-m)/m}}(0).$$

From $C^{1,\tau}$ regularity results in the ball $B_1(0)$ we would conclude that

$$\sup_{|x|<1} \|\nabla w_n\| < C$$

for a certain $C > 0$. (See for example [6].) It would follow by the Mean Value Theorem that

$$1/2 = w_n(0) - w_n\big(S_n^{(p+1-m)/m}(\widetilde{x}_n - x_n)\big) \leqslant \sup_{|x|<\delta_n S_n^{(p+1-m)/m}} \|\nabla w_n\| \widetilde{\delta}_n S_n^{(p+1-m)/m} \leqslant C \widetilde{\delta}_n S_n^{(p+1-m)/m},$$

which is impossible. □

Lemma 3.5. *Suppose $x_n \to x_0 \in \Omega$. Then there exists an $\varepsilon > 0$ such that, for all $\gamma > N(p+1-m)/m$, passing to a subsequence, we have*

$$\int_{B_\varepsilon(x_n)} u_n^\gamma \to \infty.$$

Proof. Taking a subsequence if necessary, we can suppose that $\lim \widetilde{\delta}_n = \widetilde{\delta}$. Thus there are just two possibilities as follows:

Case 1. If $\widetilde{\delta} = 0$, then take $\varepsilon > 0$ so that, for n sufficiently large, we have $\widetilde{\delta}_n < \varepsilon < \delta_n$. In this case, we have

$$\int_{B_\varepsilon(x_n)} u_n^\gamma \geqslant \int_{B_{\widetilde{\delta}_n}(x_n)} u_n^\gamma \geqslant C \widetilde{\delta}_n^N S_n^\gamma \geqslant C\big(c S_n^{N(m-1-p)/m} S_n^\gamma\big)^{1/\gamma} \to \infty.$$

Case 2. If $\widetilde{\delta} > 0$, then take $\varepsilon > 0$ so that, for n large, we have $\widetilde{\delta}_n > \varepsilon$. In this case, we have

$$\int_{B_\varepsilon(x_n)} u_n^\gamma \geqslant C \varepsilon^N S_n^\gamma \to \infty. \qquad \square$$

Lemma 3.6. *Suppose that $x_n \to x_0 \in \partial\Omega$. Then there exist $\varepsilon > 0$ and a sequence $(y_n)_n$ in Ω such that, for all $\gamma > N(p+1-m)/m$, passing to a subsequence, we have*

$$\int_{B_\varepsilon(y_n)} u_n^\gamma \to \infty. \tag{3.6}$$

Proof. Let $z_n \in \partial\Omega$ be such that $\delta_n = d(x_n, z_n)$. We denote the unit exterior normal of $\partial\Omega$ at z_n by ν_n. Fix ε sufficiently small, and take $y_n = z_n - \varepsilon \nu_n$. It will suffice, taking subsequences if necessary, to divide the proof into two cases, as follows.

Case $u_n(y_n) \geqslant S_n/2$, *for all n*. First define

$$\widetilde{\delta}_n = \sup\big\{\delta\colon\ x \in B_\delta(y_n)\ \Rightarrow\ u_n(x) > S_n/4\big\}.$$

Consider the sequence defined by the normalized functions

$$w_n(x) = S_n^{-1} u_n\big(y_n + S_n^{(m-1-p)/m} x\big).$$

As in the proofs of Lemmas 3.3 and 3.4, we may show that there exists a $c > 0$ so that, for n sufficiently large, we have

$$0 < c < \widetilde{\delta}_n S_n^{(p+1-m)/m}.$$

Thus

$$\int_{B_\varepsilon(y_n)} u_n^\gamma \geqslant \int_{B_{\widetilde{\delta}_n}(y_n)} u_n^\gamma \geqslant C\widetilde{\delta}_n^N S_n^\gamma \geqslant C\big(c S_n^{N(m-1-p)/m} S_n^\gamma\big)^{1/\gamma} \to \infty.$$

Case $u_n(y_n) < S_n/2$, *for all* n. First, we note that the function u_n satisfies

$$-\Delta_m u_n = f(x, u_n, \nabla u_n), \quad \text{for } x \in A_n$$

where $A_n = B_{\varepsilon-\delta_n}(x_n) \cap \{x/(x - x_n)\cdot \nu_n \leqslant -\frac{|x-x_n|}{2}\}$.

Now define

$$\widetilde{\delta}_n = \sup\left\{\delta\colon\ x \in B_\delta(x_n) \text{ and } (x - x_n)\cdot \nu_n \leqslant -\frac{|x - x_n|}{2} \ \Rightarrow\ u_n(x) > S_n/2\right\}.$$

Clearly, $\widetilde{\delta}_n$ is well defined, and we have

$$0 < \widetilde{\delta}_n < d(x_n, y_n) = \varepsilon - \delta_n.$$

As in the proof of Lemma 3.4, we can use the $C^{1,\tau}$ regularity to the functions w_n in the sets $B_1(0) \cap \{x/x\cdot \nu_n \leqslant -\frac{|x|}{2}\}$ to conclude that

$$0 < c < \widetilde{\delta}_n S_n^{(p+1-m)/m}.$$

Define $B_n = B_{\widetilde{\delta}_n}(x_n) \cap \{x/(x - x_n)\cdot \nu_n \leqslant -\frac{|x-x_n|}{2}\}$. A simple computation shows that $B_{\widetilde{\delta}_n}(x_n) \subset B_\varepsilon(y_n)$. Therefore,

$$\int_{B_\varepsilon(y_n)} u_n^\gamma > \int_{B_{\widetilde{\delta}_n}(x_n)} u_n^\gamma > \int_{B_n} u_n^\gamma \geqslant C\widetilde{\delta}_n^N S_n^\gamma \geqslant C\big(c S_n^{N(m-1-p)/m} S_n^\gamma\big)^{1/\gamma} \to \infty. \qquad \square$$

Now we can finish the proof of Theorem 3.2. Since $p < N(m-1)/(N-m)$, we can choose γ so that $N(p+1-m)/m < \gamma < N(m-1)/(N-m)$. Assume $x_n \to x_0 \in \Omega$ (the case $x_n \to x_0 \in \partial\Omega$ is proved in the same way). By Lemmas 2.1 and 3.5 we would have

$$\inf\big\{u_n(x)/x \in B_{\varepsilon/2}(x_n)\big\} \geqslant C\varepsilon^{-N/\gamma}\|u_n\|_{L^\gamma(B_\varepsilon(x_n))} \to \infty. \tag{3.7}$$

On the other hand, by Lemma 2.2 we would obtain

$$C\varepsilon^N\big(\inf\big\{u_n(x)/x \in B_{\varepsilon/2}(x_n)\big\}\big)^s \leqslant \int_{B_{\varepsilon/2}(x_n)} u_n^s \leqslant C\varepsilon^{(N-ms)/(q+1-m)}$$

for $s \in (0, q)$. But this is impossible by inequality (3.7). $\square$

4. Existence result

In this section, we use the a priori estimate obtained in the preceding section to establish an existence result. We state a result, due to Azizieh and Clément [1], which is an extension of the classical Rabinowitz bifurcation.

Lemma 4.1. *Let $\mathbb{R}^+ := [0, +\infty)$ and $(E, \|\cdot\|)$ be a real Banach space. Let $G : \mathbb{R}^+ \times E \to E$ be continuous and mapping bounded subsets on relatively compact subsets. Suppose moreover G satisfies*

(a) $G(0, 0) = 0$,
(b) *there exists $R > 0$ such that*:
  (i) $u \in E$, $\|u\| \leqslant R$ *and* $u = G(0, u)$ *implies* $u = 0$,
  (ii) $\deg(Id - G(0, \cdot), B(0, R), 0) = 1$.

Let J denote the set consisting of the solutions of the problem

$$(\mathfrak{P}) \quad u = G(t, u)$$

in $\mathbb{R}^+ \times E$. Let $\mathfrak{C}$ denote the component (closed connected subset maximal with respect to inclusion) of J to which $(0,0)$ belongs. If

$$\mathfrak{C} \cap \big(\{0\} \times E\big) = \big\{(0,0)\big\},$$

then $\mathfrak{C}$ is unbounded in $\mathbb{R}^+ \times E$.

The following is our existence result.

Theorem 4.2. *Suppose that condition (1.2) is satisfied and that*

$$\lim_{u\to 0} \frac{f(x,u,\eta)}{u^{m-1}} = 0 \tag{4.8}$$

where this limit is uniform with respect to the remaining variables x, η. Then there exists a positive solution of Problem (1.1).

Proof. Let $S : C^1(\overline{\Omega}) \to C(\overline{\Omega})$, given by $S(u) = f(x,u,\nabla u)$. From the continuity of f we conclude that N is continuous.

Also, we consider $T : C(\overline{\Omega}) \to C^{1,\rho}(\overline{\Omega})$ defined so that if $v \in C^{1,\rho}(\overline{\Omega})$, then $T(v)$ is the unique weak solution of the problem

$$\begin{cases} -\Delta_m u = v, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega. \end{cases}$$

It is well known that the function T is continuous and maps bounded sets into bounded sets (see for instance Lemma 1.1 in [1]). From the compactness of the inclusion $C^{1,\rho} \hookrightarrow C^1$ we have that $K := T \circ S : C^1(\overline{\Omega}) \to C^1(\overline{\Omega})$ is compact.

Let $G(t,u) := T(S(u) + t)$, then $G : \mathbb{R}^+ \times C^1(\overline{\Omega}) \to C^1(\Omega)$ is compact. Now, we will verify the hypotheses of Lemma 4.1. It is clear that $G(0,0) = 0$. On the other hand, we will show that if u is a nontrivial solution of the problem

$$\begin{cases} -\Delta_m u = \lambda f(x,u,\nabla u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{4.9}$$

where $\lambda \in [0,1]$. Then $\|u\|_{C^1} > R > 0$. In fact, it follows from hypothesis (4.8) that, given $\varepsilon > 0$ there exists a constant $\rho > 0$ such that, for all $0 \leqslant u \leqslant \rho$, and $0 < |\eta| < \rho$ we have

$$f(x,u,\eta) \leqslant \varepsilon u^{m-1}. \tag{4.10}$$

Now, multiplying (4.9) by u, integrating over Ω the equation obtained and applying the Hölder's and Poincare's inequalities, we have that for $\|u\|_{C^1} \leqslant \rho$,

$$\int_\Omega |\nabla u|^m = \lambda \int_\Omega f(x,u,\nabla u)u \leqslant \varepsilon\lambda \int_\Omega u^m \leqslant \varepsilon C\lambda \int_\Omega |\nabla u|^m$$

which implies that $\|u\|_{C^1} > R > 0$ for some small enough R. Thus part (i) of the condition (b) is satisfied, and from homotopy proprieties of the degree we have that part (ii) of condition (b) is satisfied. In addition, the solutions of $u = G(t,u)$ are bounded by Theorem 3.2. Therefore, applying Lemma 4.1 the proof follows. $\square$

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